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The applied effectiveness of interpolation algorithm on vertical electrical sounding data processing by N transformation method

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ABSTRACT

N transformation is a new method in the system of vertical electrical sounding (VES) curve transformation methods such as Petrovski method (P), Zhondy method (Z), Petrovski–Zhondy method (PZ). The VES curves and the geoelectrical sections which are processed by this method had better resolution and locality property than the methods P, Z, PZ. Mathematical essence was to transform apparent resistivity values to changing resistivity ones and conventional depths to changing ones by expressions containing derivative operations. The derivative operations were calculated in the same numerical method as in the P, Z, PZ. However, numerical processing has some limitations, for example, large error, complex computational programming, not allowing extrapolation of neighboring points. Therefore, the effectiveness of the N transformation method was reduced. To overcome these disadvantages, the author has used the interpolation algorithm to calculate the derivative operations by continuous derivatives. Results applied by this method are better than by numerical method.

1. Introduction

There are some ways to categorise electrical exploration methods. One of the most common subdivisions is based on geological environmental parameters. By this way, these methods may be subdivided into two main groups: one group is concerned with themeasurement of resistivity, or conductivity of rocks; the other one is concerned with themeasurement of their capacitance. The vertical electrical resistivity sounding method (VES method) is a resistivity method which can be applied commonly to studying variations in resistivity with depth (Mohamed, 1975). The apparent resistivity values are used in both 1D, 2D and 3D VES data processing. However, the results of processing, until recently, were limited to show the parameters of resistivity and the thickness of layers or studied objects (Nguyễn Trọng Nga, 2005 and Dương Văn Huyên, 2009). The transformation methods were established to enhance the resolution and localization of geoelectrical section (Nguyễn Trọng Nga, 2007).

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N-transformation method was introduced by (Nguyễn Trọng Nga, Trương Thị Chinh, 2016). The applied results in VES data showed that the apparent VES curves and geoelectrical sections changed by this method had the highest resolution and localization in the system of transformation methods of VES curve (Trương Thị Chinh, 2017). The interpolation algorithm was used to eliminate the limitations of the numerical processing method, therefore ensured the effectiveness of the N-transformation method.

2. N- transformation method

2.1. N- transformation method

The N-transformation method was constructed two formulas: one formula for the change in resistivity and another formula for calculating the depth. The changed formula of resistivity was developed from the Petrovski – Zhondy transformation to improve the resolution of the VES curves as well as geoelectrical sections (Nguyễn Trọng Nga, Trương Thị Chinh, 2016). The formula was as follows:

$$\rho_N(z_N)=\begin{cases}\dfrac{\rho_k(r)}{\left(1-\dfrac{\partial\lg\rho_k(r)}{\partial\lg r}\right)^2} & when\ \rho_k(r_{i+1})\le\rho_k(r_i)\\ \rho_k\left(1+\dfrac{\partial\lg\rho_k(r)}{\partial\lg r}\right)^2 & when\ \rho_k(r_{i+1})>\rho_k(r_i)\end{cases}\quad(1)$$

where $\rho_N(z_N)$ is the changed resistivity; $\rho_k(r)$ is the apparent resistivity at the size r=AB/2.

The formula of changed depth was studied by describing the effect of anisotropy in the layering environment which were not previously mentioned in the *P, Z, PZ* methods. Because of the anisotropy, the resistivity in the parallel direction and the perpendicular direction to layer surface are different. This anisotropy was characterized by an anisotropy coefficient: The anisotropic coefficient at each VES point was given by the formula:

$$\lambda=\begin{cases}\left(1-\dfrac{\partial\lg\rho_k}{\partial\lg r}\right)^2 & when\ \rho_k(r_{i+1})\le\rho_k(r_i)\\ \left(1+\dfrac{\partial\lg\rho_k}{\partial\lg r}\right)^2 & when\ \rho_k(r_{i+1})>\rho_k(r_i)\end{cases}\quad(2)$$

Due to the variable depth was associated with conventional depth by: $z_N=\alpha r=r/\lambda$, so:

$$z_N=\begin{cases}\dfrac{r}{\left(1-\dfrac{\partial\lg\rho_k}{\partial\lg r}\right)^2} & when\ \rho_k(r_{i+1})\le\rho_k(r_i)\\ \dfrac{r}{\left(1+\dfrac{\partial\lg\rho_k}{\partial\lg r}\right)^2} & when\ \rho_k(r_{i+1})>\rho_k(r_i)\end{cases}\quad(3)$$

The combination of formulas (1) and (3) obtained the complete equation of the N transformation method. The initial results showed that the curves $\rho_N(z_N)$ and $\rho_N(x_N,z_N)$ sections have higher resolution and better localization than PZ method (Trương Thị Chinh, 2017).

2.2. The numerical method in N transformation

The measurement data at a VES site was discrete data consist of the conventional depth r=AB/2 (AB is the distance between two transmitter electrodes) and the apparent resistivity value ρ_k. The function showed the relationship between the apparent resistivity value and the conventional depth as a discrete function $\rho_{ki}=f(r_i)$. When transforming the VES curves, derivatives were calculated by numerical method. Numerical derivatives were calculated by some methods in which the three-point derivative (center point derivative) method was commonly applied.

To calculate the transformed resistivity value ρ_N according to the formula (1) and the changed depth z_N in formula (3), we need to calculate the derivative as

$$\left.\frac{\partial\lg\rho_k(r)}{\partial\lg r}\right|_{r_i}\quad(4)$$

By the numerical method, when there are n value pairs of ρ_k and r, the derivative formula is calculated by the three-point derivative at each AB size as follows:

+ At the beginning points r_o and the end ones r_n, derivatives were calculated by the formula:

$$\left.\frac{\partial\lg\rho_k(r)}{\partial\lg r}\right|_{r_o}=\frac{\lg\rho_k(r_1)-\lg\rho_k(r_o)}{\lg(r_1)-\lg(r_o)}\quad(5)$$

$$\left.\frac{\partial\lg\rho_k(r)}{\partial\lg r}\right|_{r_n}=\frac{\lg\rho_k(r_n)-\lg\rho_k(r_{n-1})}{\lg(r_n)-\lg(r_{n-1})}\quad(6)$$

+ At middle points, we have:

$$\left.\frac{\partial\lg\rho_k(r)}{\partial\lg r}\right|_{r_i}=\frac{\lg\rho_k(r_{i+1})-\lg\rho_k(r_{i-1})}{2h}+0(h)^2\quad(7)$$

Where $h = \lg(r_{i+1}) - \lg(r_i)$ was called discrete step, $0(h)^2$ was infinitely small and proportional to h. It was often considered to be zero in transformations. So:

$$\left.\frac{\partial \lg \rho_k(r)}{\partial \lg r}\right|_{r_i} \approx \frac{\lg \rho_k(r_{i+1}) - \lg \rho_k(r_{i-1})}{2h} \quad (8)$$

Thus, the derivatives were calculated non-uniform at all points. Besides, the results of the derivative operations always existed the rounding error when omitted $0(h)^2$. With the same function, the smaller the h was, the smaller the error was and vice versa. Substituting the derivative results into formulas (1) and (3) leaded to the indirect error. Therefore transformation results were incorrectandthe effectiveness of the transformed method N wasn't guaranteed.

2.3. Interpolation algorithm in N transformation method

Suppose that we have the following measurement data table:

Table 1. The measurement data table.

X	x_1	x_2	x_3	x_4	x_5	x_6	......	x_{n-1}	x_n
Y	y_1	y_2	y_3	y_4	y_5	y_6	......	y_{n-1}	y_n

The interpolation method was done by finding a continuous functions *f* such that their value domain contains points $(x_o, x_1, ..., x_n)$. These functionswere chosen so that they were the best to approximate the existing pair dataset: (x_o,y_o), (x_1,y_1),…,(x_n,y_n). They was called the approximation function. The functions were countless while discrete data sets had only one because there are many interpolations, each interpolation had its own rules and conditions. Polynomial interpolation was one of the most common methods. The real data at each VES point was approximated by a polynomial of greatest degree (n-1). This polynomial was called interpolation polynomial. Polynomial interpolation included many methods as Lagrangeinterpolation, Newton interpolation, spline interpolation, Interpolation by the least - squares method (Nguyễn Xuân Huấn, 2004). In the paper, the author chose the least squares interpolation. By this method, the data was approximated by the function y=f(x) which is polynomial of degree (n-1):

$$y = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_o$$

Coefficient sets a_{n-1}, a_{n-2},…, a_1, a_o were selected to satisfy the minimum squared condition. This means

$$S = \sum_{k=1}^{n}(a_{n-1}x_k^{\,n-1} + a_{n-2}x_k^{\,n-2} + ... + a_1 x_k + a_o - y_k)^2$$

reach the smallest value.

In the VES data processing, x was replaced by lg(r), y was replaced by lg(ρ_k). After approximating the discrete function to the approximated polynomial of degree (n-1), we obtained a continuous function lg(ρ_k) = f(lg(r)). So, it was easy to calculate the derivative $\partial(\lg\rho_k)/\partial(\lg r)$. As the derivative of an algebraic polynomial:

$$\frac{\partial \lg \rho_k}{\partial \lg r} = (n-1)a_{n-1}(\lg r)^{n-2} + (n-2)a_{n-2}(\lg r)^{n-3} + ... + a_1 \quad (9)$$

After interpolation processing, the function lg(ρ_k) = f(lg(r)) was a continuous function so its derivative was also a continuous derivative. Substituting formula (9) into formula (4) we obtained the formula of N transformation method as follows:

$$\rho_N(z_N) = \begin{cases} \dfrac{\rho_k(r)}{\left(1 - \dfrac{\partial \lg \rho_k(r)}{\partial \lg r}\right)^2} & when\ \dfrac{\partial \lg \rho_k(r)}{\partial \lg r} \le 0 \\ \rho_k\left(1 + \dfrac{\partial \lg \rho_k(r)}{\partial \lg r}\right)^2 & when\ \dfrac{\partial \lg \rho_k(r)}{\partial \lg r} > 0 \end{cases} \quad (10)$$

$$z_N = \alpha r = \begin{cases} \dfrac{r}{\left(1 - \dfrac{\partial \lg \rho_k(r)}{\partial \lg r}\right)^2} & when\ \dfrac{\partial \lg \rho_k(r)}{\partial \lg r} \le 0 \\ \dfrac{r}{\left(1 + \dfrac{\partial \lg \rho_k(r)}{\partial \lg r}\right)^2} & when\ \dfrac{\partial \lg \rho_k(r)}{\partial \lg r} > 0 \end{cases} \quad (11)$$

3. The applied effectiveness of interpolation algorithm on VES data processing by N transformation method

3.1. The applied effectiveness on the theoretical curves

Considered the three-layer curve H-1/4-4-∞ with environmental parameters:

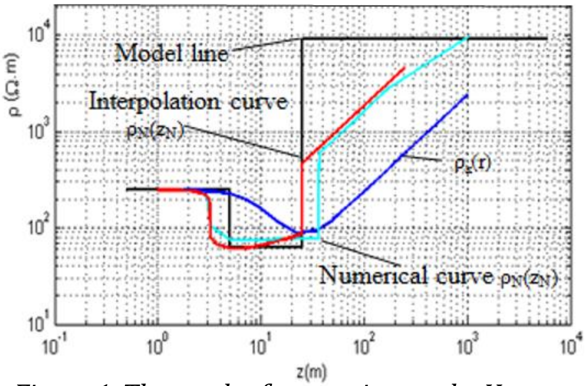

Figure 1. The result of processing on the H curve.

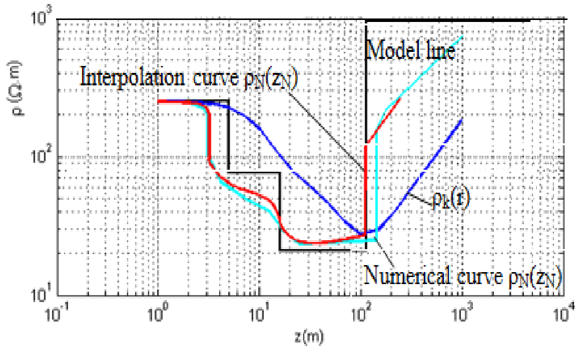

Figure 2. The result of processing on the QH curve.

$\rho_1 = 248\,\Omega\text{m}; \text{h}_1 = 5\text{m}$; $\rho_2 = 62\,\Omega\text{m}; \text{h}_2 = 20\text{m}$; $\rho_3 = \infty; \text{h}_3 = \infty$. The numerical method and the interpolation method were used in N transforming and results were shown in Figure 1.

According to results of figure 1, in the three curves: the apparent resistivity curve $\rho_k(r)$, the changed curve $\rho_N(z_N)$ by numerical method(the numerical curve) and the changed curve $\rho_N(z_N)$ by interpolation algorithm (the interpolation curve), the interpolation curve was the closest to the model line. Especially, in the last curve branch, while the digital curve was shifted to the right of the model line, the interpolation curve was coincident with the model line. In term of extremes, the interpolation curve had a greater amplitude than the numerical curve and was approximately equal to the minimum of the model line.

Applying these methods on the QH curve of geological environment: $\rho_1 = 250\ \Omega m;\ h_1 = 5m;$ $\rho_2 = 76\ \Omega m;\ h_2 = 11m;\ \rho_3 = 21\ \Omega m;\ h_3 = 100m;$ $\rho_4 = \infty\ \Omega m;\ h_4 = \infty$. The results were displayed in figure 2.

Figure 2 showed the results of QH curve. These results also showed that N transformation curve by interpolation method was the closest to the model line Also, the inflection point and extreme were clarity. Briefly, the results of the two theoretical resistivity curves prove that the interpolation curves were not only higher in resolution but also had better localization than numerical one. Besides, they reflected the depth of layers more accurately than the numerical method.

3.2. The applied effectiveness on the measured curve

N transformation method was applied using interpolation algorithm in data processing to investigate the distribution and thickness of weathered zone on T1 line - X region. As a result of the geological drilling inthis area, the T1 geo-layered model consists of the following layers:

Layer 1: The cover layer consists of a mixture of clay - sand and organic material with boulders, thickness from is a few centimeters to 1 meter, the value of resistivity is $\rho_1 = 700 - 1500\Omega m$

Layer 2: The ruin layer consists of a mixture of clay and gravel, itsthicknessis from a few meters to 10meters, the resistivity is $\rho_2 = 300 - 1200\ \Omega\text{m}$

Weat Layer 3: The weathered zone which is unevenly distributed. The mainly granite rock is weathered strongly, medium and light. There are places fully weathered. The thickness is around ten of meters to 100m and the resistivity is . Granite which is weathered completely can be less than $600\Omega m$

Layer 4: The original stone layer has very large resistivity $\rho_4 > 4000\Omega m$

Thus, at the studied area, the VES curves are of QH or HA type. They were changed by N transformation method using both numerical and interpolation methods. The results were described in Figure 4. The curves were compared, the apparent resistivity curve c was seen as 4-layers curve QH but not clearly because the inflection points and extremes are unclear.

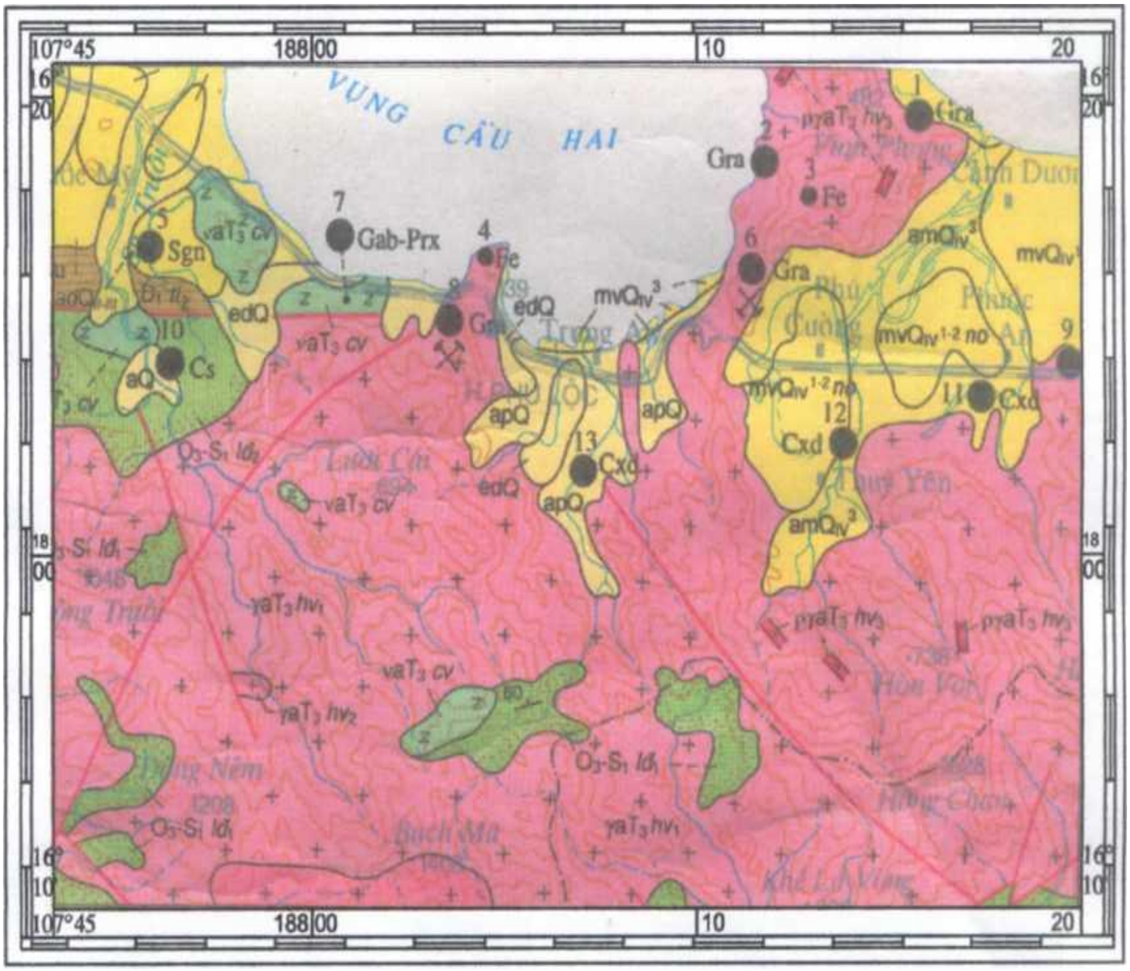

Figure 3. Geological map of studied area.

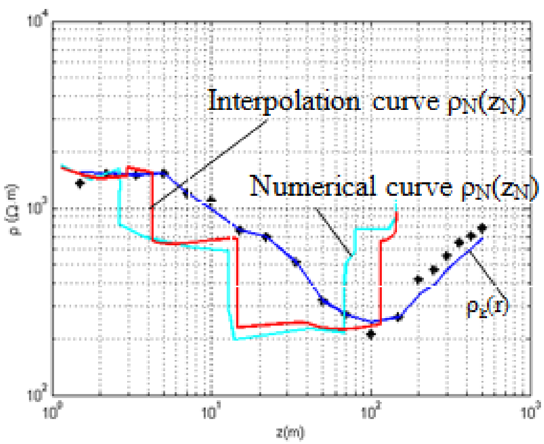

Figure 4. The result on actual QH curve.

The curves $\rho_N(z_N)$ treated by numerical and interpolation methods included clearer inflection points and extremes express the number of layers better. In which the interpolation curve most accurately reflected 4-layers curve QH. In addition, the layer boundaries shown in the interpolation curve were more appropriate than the numerical curve due to the approximate boundary at the inflection points and the extremes of the apparent resistivity curve $\rho_k(r)$. Besides, the thickness of the layers shown on the interpolation curve was more appropriate: the resistivity of the first layer was about 1400Ωm, its depth was 4m – respectively high resistivity layer; the second layer had approximate resistivity 700Ωm, 11m deep - the ruin layer consisted of a mixture of clay and gravel; the third layer had approximate resistivity 230Ωm, 95m deep– the granite layer was weathered strong and completely; the fourth was very high resistivity layer (1400Ωm). To demonstrate the effectiveness of the interpolation algorithm in the N-transform method, the author applied this method for the whole measurement line T1 in the X region. The results were shown in Figure 5.

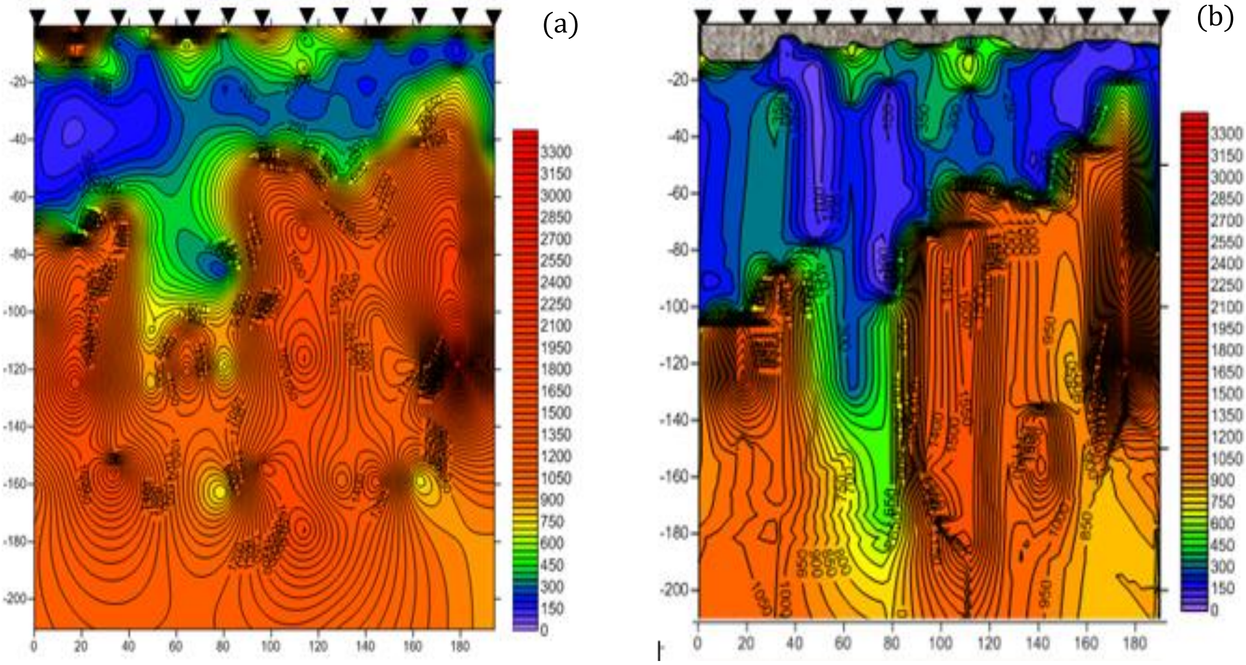

Figure 5.(a)The section changed by numerical method; (b)The section changed by interpolation method.

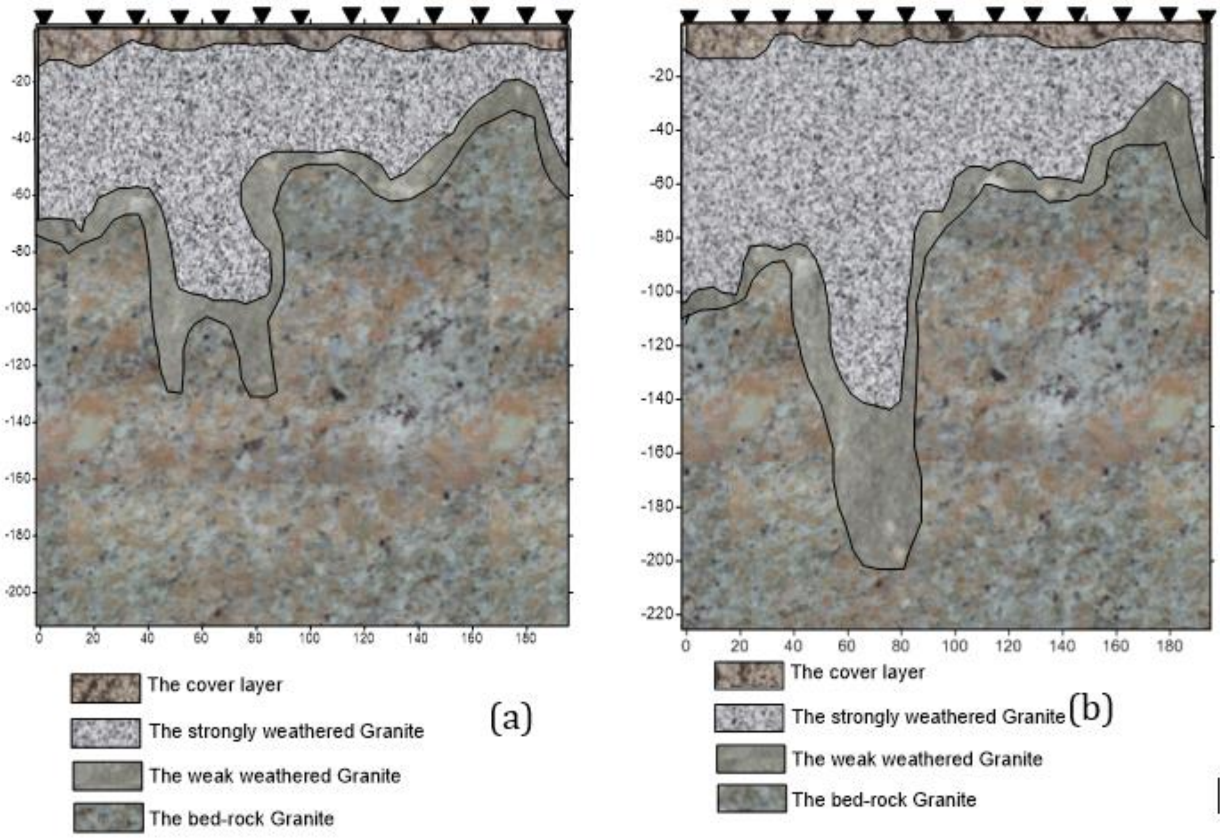

Figure 6.(a) The geological section interpreted from the section changed by numerical method; (b) The geological section interpretedfrom the section changed by interpolation method.

We can see that $\rho_N(x_N,z_N)$ sections processed by numerical and interpolation methods both showed relatively accurate the cliff complex surface of granite. In addition,there was a strong weathered zone at the beginning of the line, fall down at the end. This zone developed both horizontally and vertically direction (site 40 to site 80) on these sections. However, the section changed by interpolation method (Figure 5b) showed that the weathered layer was more clearly and thickness was greater than one transformed by numerical method (Figure 5a) at most sites. Transformation sections were interpreted and shown in Figure 6.

As the results in Figure 6, on both geological sections, the granite layers were weathered with different level, more strong at the beginning and gradually decreased to the end of the line. From site 40 to site 80 they grew in depth.

On the interpretation section from the section changed by the numerical method (Figure 6a), the depth of strong weathered granite layer changed from 15m to 60m and reached 90m at sites between 40 and 80, the apparent resistivity was less than 500Ωm. Below this layer was a weakly weathered granite layer with the medium thickness from 5m to 10 m and reached 40m at sites between 50 and 80. The apparent resistivity was 500 – 700 Ωm. The bottom layer was granite bed-rock with high resistivity.

On the interpretation section from the section changed by interpolation method (Figure 6b), the thickness of strong weathered granite layer changes from 20m to100m, between site 40 and site 80, this figure was 140m; the apparent resistivity was less than 500Ωm weakly weathered granite layer with the medium thickness being from 5m to 10 m and reached 60m at sites between 50 and 80; the apparent resistivity was 500 – 700 Ωm. The bottom layer was granite bed-rock with high resistivity.

Briefly, the interpreted result from the section changed by interpolation method displayed the thickness of the weathered granite layerto be more suitable with the result of the geological survey than one by numerical method.

4. Conclusion

The interpolation algorithm was used to ensure the effectiveness of the N-transformation method in the VES data processing. The effectiveness of interpolation was demonstrated through the applied results on the theoretical apparent resistivity curve and the measurement apparent resistivity one:

The results on the theoretical curve showed that the changed VES curves $\rho_N(z_N)$ by interpolation method had higher resolutions, better localization, so reflects more accurately the boundary and thickness of layers than the curves $\rho_N(z_N)$ transformed by numerical method.

The measurement VES curve $\rho_N(z_N)$ changed by interpolation method also had higher resolutions, better localization. This allows determining relatively the number of geological layers, boundaries and the thickness of layers compared to drilling data. Meanwhile, the changed curve $\rho_N(z_N)$ by numerical method also gave high resolution but information about the number of layers, the thickness of layers was not accurate.

The changed section $\rho_N(x_N,z_N)$ using interpolation method expressed clearly the topography of the class, the particularly concave topography of the bed-rock. In addition, the interpolation section reflected more accurately the boundary and the thickness of layers than the one treated numerically. The thickness of the layers was relatively accurate compared to that of the geological drilling documentation.

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